A New Look at Keynesian Weight of Evidence

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Abstract

John Maynard Keynes's concept of the weight of evidence is one of the most enduring—and mysterious—features of Keynes's epistemology. One popular interpretation of Keynesian weight comes courtesy of Jochen Runde, who argues that weight is best construed as the ratio of the amount of known relevant evidence to the amount of total relevant information. Runde's interpretation aims to establish a strong link between Keynesian weight and rational confidence, which in turn is meant to bolster the connection between Keynesian weight of evidence and Keynes's later economic work. Despite this advantage, Runde's interpretation also implies the existence of significant inconsistencies in Keynes's account of weight of evidence. As a consequence, Runde's interpretation paints Keynes as particularly confused about Keynesian weight of evidence. In this work, I provide a distinction between estimated Keynesian weight and actual Keynesian weight in order to preserve the desirable features of Runde's interpretation while simultaneously resolving the textual inconsistencies that arise from that interpretation. The upshot is a novel interpretation that makes Keynesian weight less ambiguous, more consistent with the rest of Keynes's epistemology, and better-situated for future research.

Keywords: weight of evidence, John Maynard Keynes, Keynesian probability, unknown unknowns, uncertainty, Paradox of Ideal Evidence

1 Introduction

Suppose that at your next visit to the doctor, the doctor brings two vaccines into the office and says, "The third-party testing indicates that these two vaccines are equally effective at protecting patients from this year's flu. Additionally, based on my records of patient-reported side effects, both vaccines have a 5% (classical) probability of bringing about mild side effects that last for 24 hours and a 95% (classical) probability of no side effects at all. Which vaccine would you like me to administer?" At this point, I imagine you are indifferent to the two vaccines. But the doctor goes on to say, "I have administered *VaccineA* to 20 patients and 1 patient reported side effects. On the other hand, I have administered *VaccineB* to 10,000 patients, and 500 of them reported side effects." Are you still indifferent between the two vaccines?

Whenever you encounter a decision like the one above, you encounter the need to analyze your evidence. In the example above, the doctor's probability statements tell you about the balance of the evidence with respect to vaccine side effects. While probability statements clearly tell you something about your evidence, the above example shows that probability statements fail to tell the whole story. In John Maynard Keynes's 1921 book, *A Treatise on Probability*, Keynes outlined a concept—now called "Keynesian weight" or simply "weight of evidence"—that can potentially supplement probability in summarizing evidence. The idea behind Keynesian weight is to measure the amount of relevant evidence backing a probability statement. Michael Titelbaum notes that weight of evidence was later linked to the resiliency of a probability statement (Titelbaum, 2022, 514–5).[1] The utility of a concept such as Keynesian weight has been recognized at least as far back as Charles Sanders Peirce, who said, "To express the proper state of our belief not *one* number but *two* are requisite, the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based" (Peirce, 1992, 160).[2] Even though Keynesian weight ignited extensive debate on the nature of uncertainty, the concept remains obscure.

The existence of some confusion regarding Keynesian weight comes as no surprise to readers of *A Treatise on Probability*. After all, Keynes begins the chapter on weight by saying, "The question to be raised in this chapter is somewhat novel; after much consideration, I remain uncertain as to how much importance to attach to it," and he goes on to say, "A little reflection will probably convince the reader that this is a very confusing problem" (Keynes, 1921, 78, 85). Exacerbating this difficulty, Jochen Runde argues that there are at least two distinct ways in which Keynes discusses weight (Runde, 1990, 279). According to Runde, the "relative" conception—in which weight is the ratio of the amount of known relevant evidence to the amount of total relevant information—is the best conception of weight presented by Keynes (Runde, 1990, 283). On this point, I agree with Runde. That said, Runde's interpretation results in many of Keynes's remarks becoming inconsistent, which seems to make Keynesian weight hopelessly ambiguous. If Runde is correct, then Keynes was even more confused

[1] Using Keynesian weight of evidence to measure the dependability, stability, or resiliency of a probability statement is controversial. See Hamer (2012) for a skeptical position, and see Bradley (2019), Section 2.3 for more discussion.

[2] Like Keynes, Peirce's use of the word "knowledge" does not always refer to justified true belief. Peirce and Keynes frequently use the word "knowledge" and its cognates in a loose sense. Peirce's "The Probability of Induction" was published in 1878 and is the earliest direct treatment of "Keynesian" weight of evidence of which I am aware. Although Keynes cites Peirce in *A Treatise on Probability*, Keynes does not connect Peirce to any considerations of weight. Instead, Keynes's treatment of weight references an 1890 book review by Meinong, an 1892 article by A. Nitsche, and some suggestions in Czuber's 1903 work titled *Wahrscheinlichkeitsrechnung*. In C.D. Broad's 1922 review of *A Treatise on Probability*, Broad says, "I do not know of any other writer who has raised [the question(s) posed by weight] except myself in the chapter on Causation in *Perception, Physics, and Reality*; though I do not doubt that Mr. [W.E.] Johnson has an elaborate treatment of it up his sleeve" (Broad, 1922, 78). Broad's analysis in *Perception, Physics, and Reality* appears fairly similar to Keynes's own analysis (Broad, 1914, 150–5).

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than Keynes admits. It is worthwhile to consider the true extent of Keynes's confusion; I maintain that Keynes was clearer-headed than Runde's interpretation makes him seem.

In what follows, I propose a novel interpretation of Keynesian weight. To that end, I first provide some necessary background on Keynesian weight and Runde's interpretation. Ultimately, I argue that by distinguishing between estimated and actual Keynesian weight, we can retain the primary features of Runde's interpretation while simultaneously alleviating the textual inconsistencies created by that interpretation. More specifically, I claim that reconsidering the meaning of "relevant ignorance" (i.e., relevant evidence of which an agent is unaware) leads to an interpretation of relative Keynesian weight that thoroughly aligns with Keynes's remarks. The interpretation I offer better situates Keynesian weight within Keynes's own work while also illuminating directions for future research into the applicability of Keynesian weight.

2 The Basics of Keynesian Weight

My goal in this section is to provide a brief introduction to Keynesian weight. To accomplish that goal, it is helpful to begin with a broad outline of the Keynesian approach to probability. Next, I introduce the core idea behind Keynesian weight while also highlighting some connections between Keynesian probability and weight. With those elements in place, I conclude this section with some consideration of the reasons why contemporary philosophers might care about Keynesian weight.

2.1 The Tie to Keynesian Probability

Keynes understood probability to be "a general theory of logic, the purpose of which is a universal explanation of rational inference" (O'Donnell, 1989, 31). For Keynes, a probability refers to a two-place, logical relationship that exists between the premises and conclusion of an argument. Keynes compares probability to distance. The distance between Salt Lake City and Denver is a two-place relation that tells you how far you must travel to get from one city to the other. Likewise, a Keynesian probability is a two-place relation that tells you the extent to which the truth of the conclusion of an argument is entailed by the premises of that argument. Although a Keynesian probability is fundamentally a logical relationship between the premises and conclusion of an argument, it can also be used to support degrees of rational belief that a person should place in the conclusion of the argument, given the premises of the argument (Keynes, 1921, 3–4, 7). As such, a deductively valid argument (i.e., an argument with premises that logically entail the conclusion) possesses a probability relation of 1.0, which warrants full belief in the conclusion of the argument given the premises of the argument. Because Keynesian probability is a two-place relation between premises and a conclusion, it does not make sense to speak of Keynesian probabilities of isolated propositions (Keynes, 1921, 6).

With minor changes to Keynes's notation, the probability of an argument can be expressed as:

$$\Pr(c|e),$$

where c is the conclusion of the argument, and e represents the premises (or evidence) of the argument. As Keynes explains, the above notation ensures that our formal representation of the informal (and inaccurate—on Keynes's picture) phrase, "The probability of c," contains explicit reference to the evidence upon which the probability is based (Keynes, 1921, 43). Consequently, Keynes's notation ensures that a probability value remains tied to the argument that generates the probability relation it expresses. One reason why the connection to arguments is important for Keynes is because Keynes says that probabilities will often be non-numerical and sometimes even non-comparable (Keynes, 1921, 36). If two arguments possess the right type of relationship to each other, then the probability values justified by those two arguments will be comparable (Keynes, 1921, 40). Keynes argues, however, that probability values will only be *numerically comparable* in special cases (Keynes, 1921, 182). We should keep Keynes's emphasis on arguments in mind as we turn to Keynesian weight.

Keynes introduces weight in Chapter VI of *A Treatise on Probability*. The second paragraph of that chapter reads as follows:

> As the relevant evidence at our disposal increases, the magnitude of the probability of the argument may either decrease or increase, according as the new knowledge strengthens the unfavourable or the favourable evidence; but something seems to have increased in either case,—we have a more substantial basis upon which to rest our conclusion. I express this by saying that an accession of new evidence increases the weight of an argument. New evidence will sometimes decrease the probability of an argument, but it will always increase its "weight" (Keynes, 1921, 78).

In other words, the weight of an argument provides some account of the amount of evidence used in an argument. Moreover, Keynes draws a clear distinction between the weight of an argument and the probability of that argument. James Joyce characterizes this distinction by saying, "The intuition here is that any body of evidence has both a kind of valence and a size" (Joyce, 2005, 158). The probability of an argument measures the balance of the evidence, which makes probability an excellent tool for capturing the "valence" of the body of evidence. In contrast, the weight of an argument is meant to capture the "size" of the body of evidence. As Keynes says, "[T]he weighing of the amount of evidence is quite a separate process from the balancing of the evidence for and against" (Keynes, 1921, 81). With these remarks, we have the core idea behind Keynesian weight in place. We can now see that while the two vaccines in our opening example had equivalent probabilities of side effects, the probability associated with *VaccineB* had greater weight than the probability associated with *VaccineA*.

We can also see why it was crucial to begin with some discussion of Keynesian probability. Like probability, Keynes saw weight as a logical, two-place relation between the premises and conclusion of an argument. As Rod O'Donnell says, "The point [that weight of *argument* is the fundamental concept] is further reinforced both by [Keynes's] choice of notation, that is, $V(c|e)$ rather than $V(e)$ or merely e, and by his rules for comparing weights" (O'Donnell, 1989, 68).[3] Keynes's notation shows that weight—like probability—is a relation that arises out of whole arguments. Furthermore, Keynes's explication of weight illustrates that weight of evidence is a measure of *relevant evidence*, rather than evidence generally. Of course, not all evidence will be relevant to a given conclusion, and it is only in light of a given conclusion that some evidence is relevant.[4] As a consequence, the weight of the evidence depends, at least in part, on the relationship between the premises of the argument and the conclusion of that argument. In other words, just as it does not make sense to speak of the Keynesian probability of a conclusion divorced from that conclusion's associated argument, it likewise does not make sense to speak about the "weight of evidence" divorced from the argument with which that evidence is associated.

Though originally developed within the context of Keynes's logical theory of probability, weight is still of some interest to contemporary Bayesians, particularly those focused on accounting for second-order uncertainty or the nature of evidence.[5] However, further parallels between Keynesian weight and Keynesian probability cause difficulties with respect to a fully Bayesian conception of weight. As Keynes explains of arguments, "It will often be impossible to compare their weights, just as it may be impossible to compare their probabilities" (Keynes, 1921, 79). Of course, comparisons of weight might be easy to make if we assumed that arguments with the same premises always possessed the same weights. That said, "The existence of a rule for comparing the weights of arguments such as $c|e$ and $c \wedge d|e$ is denied [by Keynes], while arguments of the form $c|e$ and $d|e$ need not always have equal weights" (O'Donnell, 1989, 69).[6] Due to the parallels between Keynesian probability and weight, the common terminological move from "weight of arguments" to "weight of evidence" risks losing some of Keynes's original meaning.[7] Any attempt to remove weight of arguments from the

[3] Throughout the paper, I replaced notation in quotations with notation that is more consistent with the rest of the paper. On the topic of notation, note that it might seem somewhat odd that the letter V was selected by Keynes instead of the letter W. Keynes, however, did not originally refer to the weight of argument as "weight," but rather as the "value of the argument" (O'Donnell, 1989, 69). It seems that Keynes switched his terminology from "value" to "weight" primarily because of the ambiguity that would arise from the use of the word "value" in this context.

[4] In this paper, I assume Keynes's "stricter and more complicated definition" of "relevant evidence" (Keynes, 1921, 60). According to that definition of relevant and irrelevant evidence (which are complementary terms), a proposition e_1 is relevant to $c|e$ if and only if there exists a proposition e_2 inferrable from the conjunction $e \wedge e_1$ but not from e alone such that $\Pr(c|e \wedge e_2) \neq \Pr(c|e)$. Although I am assuming Keynes's strict definition, I recognize that there are substantial issues with defining relevance, and I recognize that those issues play a central role in any comprehensive explication of a measure for Keynesian weight. Despite their importance, issues related to defining relevance are too far afield for my present purposes.

[5] See Joyce (2005) for one fairly recent attempt to incorporate weight into a Bayesian framework, and see Titelbaum (2022, 512–6) for discussion of Keynesian weight in relation to Bayesianism.

[6] This quote from O'Donnell draws on Keynes's discussion of three potential rules for comparing the weights of arguments, which are meant to parallel the three rules Keynes provides for comparing the probabilities of arguments. The relevant discussion appears in Sections 3 and 4 of Chapter VI of *A Treatise on Probability* (Keynes, 1921, 79–81). Keynes says, "There is, however, no rule for comparisons of weight corresponding to (iii.) above. It might be thought that $V(a \wedge b|h) < V(a|h)$, on the ground that the more complicated an argument is, relative to given premisses, the less is its evidential weight. But this is invalid"(Keynes, 1921, 80). Keynes then explains why he thinks the rule is invalid. I am grateful to an anonymous reviewer for urging me to clarify this textual link.

[7] Furthermore, Keynesian weight of arguments often gets conflated with I.J. Good and Alan Turing's similarly named, yet distinct concept of "weight of evidence." To avoid further confusion, it is vital to maintain the distinction between Keynesian weight and Good-Turing weight when discussing either concept. Thus, assume any references to "weight" in this paper refer to Keynesian weight. For more on this conflation, see Kasser (2016) and Nance (2008).

Keynesian, logical framework must reckon with the parallels between Keynesian probability and weight. With that context in mind, let us now consider why someone may wish to appropriate Keynesian weight into a modern probability framework.

2.2 Motivation for Keynesian Weight

Karl Popper's so-called "Paradox of Ideal Evidence" motivates why we might care about Keynesian weight.[8] Popper asks us to consider the probability that a coin will turn up heads on the next toss. At the start of the thought experiment, we lack any relevant information about the coin. Due to the normal appearance of the coin, we assume the probability that the coin will land heads on the next toss is 0.5 (Popper, 1980, 407). Then, Popper asks us to suppose that we find a note containing a record of statistical evidence about the coin. The statistical evidence indicates that the coin underwent a million tosses in the past, approximately half of which resulted in heads while the other half resulted in tails (Popper, 1980, 407). The statistical evidence is meant to be ideally favorable to the hypothesis that the coin is fair, so assume that the evidence is trustworthy. Given this new evidence, our updated probability estimate of the coin landing on heads on the next toss is 0.5. For Popper, the "paradox" arises from the idea that our new probability fails to incorporate the *ideally favorable evidence* in any clear way (Adler, 2002, 252).

Even though the probability remains constant between the two stages of Popper's example, "something seems to have increased in either case" (Keynes, 1921, 78). Based on Keynes's outline of weight, the second probability statement possesses higher weight than the first probability statement; although the two statements are equivalent in one sense, the second is supported by *more* relevant evidence. Keynes gives a good summary of this feature of weight when he says that holding all else equal, "It seems plain that there is some sense in which a probability founded upon more evidence is superior to one founded upon less" (Keynes qtd. in O'Donnell (1989, 67)). For Keynes, weight captures a crucial nuance about probabilities: probabilities based on more evidence often provide a preferable guide to conduct compared to probabilities based on less evidence.[9]

Consequently, developing a contemporary analogue of Keynesian weight—rather than keeping it tethered to Keynes's original probability theory—provides a promising way for Bayesians to avoid Popper's Paradox of Ideal Evidence and related problems. For instance, Margherita Harris considers various responses to Popper's Paradox of Ideal Evidence (and similar problems) as a way to explore Bayesian attempts to account for weight of evidence (Harris, 2021, 220–37). In the end, Harris leverages Jochen Runde's interpretation of Keynesian weight to conclude that "Keynes's notion of 'the weight of evidence' is an ambiguous notion since it can be understood in at least two rather different ways" (Harris, 2021, 282). In the next

[8] The "paradox" arises in the article "A Third Note on Degree of Corroboration or Confirmation," which can be found in later editions of Popper's *The Logic of Scientific Discovery*. Here, I use Popper's paradox to motivate why we might care about Keynesian weight. The paradox can also be read as a way to highlight issues with Keynes's definition of "relevant evidence." See O'Donnell (1992).

[9] Broad makes similar remarks: "Everyone will admit that we ought to prefer a probability calculated on a wider to one calculated on a narrower basis, even though the man who only had the narrower basis of knowledge had made his calculations properly" (Broad, 1914, 151).

section, I investigate this particular source of Harris's pessimism regarding Keynesian weight.[10]

3 Runde's Interpretation

In this section, my goal is to summarize Runde's interpretation of Keynesian weight. Runde's interpretation implies that Keynesian weight is ambiguous, and Runde argues that the lesser-known "relative" conception of Keynesian weight is the best conception of weight. Below, I outline the two different conceptions of Keynesian weight presented by Runde. Then, I provide an overview of textual inconsistencies that arise from Runde's preferred conception.

3.1 Sheer Weight

The most straightforward conception of Keynesian weight comes via "sheer Keynesian weight." Sheer Keynesian weight simply measures the absolute amount of relevant evidence on which a probability relation is based. This conception is sometimes called "monotonic Keynesian weight" because on this reading, a gain in relevant evidence must (by definition) increase Keynesian weight. It is impossible for sheer Keynesian weight to decrease when learning new relevant evidence. To draw an analogy to our everyday, physical notion of weight, the sheer conception of Keynesian weight treats the addition of relevant evidence similarly to the addition of mass while holding the force of gravity constant—additional mass always increases weight.

We can formally represent sheer Keynesian weight as the following:

$$V(c|e) = K.$$

Note that the above representation does not appear anywhere in Keynes's own work. Instead, Runde uses this formalization to provide a useful framework for thinking about the characteristics of sheer Keynesian weight. Above, $c|e$ represents the argument and, in keeping with Keynes's own choice of notation, V represents sheer Keynesian weight. The term K stands for "the amount of known relevant evidence," which Keynes and subsequent scholars have sometimes simply called "knowledge." In this context, knowledge does *not* refer to justified true belief in the traditional philosophical sense. Instead, knowledge means something closer to the relevant evidence that a reasoner has in their possession. Throughout the paper, I use the word "knowledge" and its cognates in the loose sense used by Keynes.

Using the above representation, we can begin to think about how new relevant evidence alters sheer Keynesian weight. Let e represent our relevant evidence (i.e., the premises of our argument) at time t_1. Let e' represent some new relevant evidence that we learn at time t_2. Since we are assuming that the evidence is relevant, we know that

$$\Pr(c|e) \gtreqless \Pr(c|e \land e'),$$

[10] Harris is generally skeptical of Bayesian attempts to account for the weight of evidence because she argues that weight of evidence causes trouble for the likelihood principle (Harris, 2021, 237). Addressing all of Harris's skeptical concerns is not the aim of this paper; instead, the paper addresses one single source of Harris's pessimism, namely the textual ambiguity of Keynesian weight.

while

$$V(c|e) < V(c|e \wedge e').$$ [11]

When we learn new relevant evidence, sheer Keynesian weight increases. The amount by which weight increases changes based on the size of our evidential gains, but any accrual of evidence leads to an increase in sheer Keynesian weight. As we will see, that is not the case for the relative conception of Keynesian weight.

3.2 Relative Weight

Runde gleans two non-monotonic conceptions of weight from Keynes's work (Runde, 1990, 280-1). Below, I show that these two conceptions are mathematical transformations of one another. Consequently, these two non-monotonic conceptions of Keynesian weight are in fact two sub-conceptions of what I call "relative weight." I address these sub-conceptions in order.

On the first page of Chapter VI, Keynes says,

> The magnitude of the probability of an argument, in the sense discussed in Chapter III., depends upon a balance between what may be termed the favourable and the unfavourable evidence; a new piece of evidence which leaves this balance unchanged, also leaves the probability of the argument unchanged. But it seems that there may be another respect in which some kind of quantitative comparison between arguments is possible. This comparison turns upon a balance, not between the favourable and the unfavourable evidence, but between the absolute amounts of relevant knowledge and of relevant ignorance respectively (Keynes, 1921, 78).

As such, Keynes's introduction of weight describes a ratio in which the amount of relevant evidence in our possession (or knowledge) forms the numerator and the amount of relevant evidence we lack (or ignorance) forms the denominator. Due to parallels to statistical odds, I call this sub-conception of relative weight "relative weight$_{odds}$" and use the notation W to represent it.

With that initial characterization in mind, consider the following formal representation of relative weight$_{odds}$:

$$W(c|e) = \frac{K}{I}.$$

Once again, note that the above representation does not appear in Keynes's work. K again denotes the amount of known relevant evidence, while the new term (I) denotes the amount of relevant evidence of which we are ignorant (i.e., the amount of unknown relevant evidence) (Runde, 1990, 280).[12] It should be clear that when $K > I$, $W > 1$; when $K < I$, $W < 1$; when $K = I$, $W = 1$; and that when $K = 0$, $W = 0$. The range of possible numerical values of W demonstrates the parallel between relative weight$_{odds}$ and statistical odds.

Later, Keynes says,

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[11] This clearly follows from Keynes's second rule for comparing weights in combination with Keynes's "stricter and more complicated definition" of relevant evidence (Keynes, 1921, 60, 80).

[12] In conversation, Steve Downes mentioned that we may possess some reason to think that the I term of this expression is infinite. If that is the case, then weight must always be equivalent to 0, which would cause some trouble for the weight-of-evidence concept. Thus, if Runde and I are correct to think that Keynes uses a relative conception of weight, then Keynes did not think that the amount of relevant evidence of which we are ignorant is infinite.

> In the present connection the question comes to this—if two probabilities are equal in degree, ought we, in choosing our course of action, to prefer that one which is based on a greater body of knowledge?... The question appears to me to be highly perplexing, and it is difficult to say much that is useful about it. But the degree of completeness of the information upon which a probability is based does seem to be relevant, as well as the actual magnitude of the probability, in making practical decisions (Keynes, 1921, 357-8).

Here, Keynes describes a measure of the "completeness of information," which Runde identifies as another version of relative Keynesian weight (Runde, 1990, 281). I call the completeness-based version of relative weight "relative weight$_{comp}$" and use the notation C_W to represent it. Runde suggests expressing relative weight$_{comp}$ as the ratio of the amount of relevant knowledge over the total amount of relevant information, where the total amount of relevant information is the sum of the amounts of relevant knowledge and relevant ignorance, respectively (Runde, 1990, 281). Accordingly, relative weight$_{comp}$ can be formally represented as:

$$C_W(c|e) = \frac{K}{(K+I)}.$$

K and I once again represent amounts of relevant knowledge and relevant ignorance. As before, the above representation is part of Runde's interpretation of Keynes and is not found in Keynes's own work. It should be clear that when $K > I$, $C_W > 0.5$; when $K < I$, $C_W < 0.5$; while $C_W = 1$ when $I = 0$; and that $C_W = 0$ when $K = 0$. Notice that the range of possible values of relative weight$_{comp}$ parallels the range of possible values of a statistical probability.

Furthermore, just as one can easily transform a statistical odds value into statistical probability value, it is straightforward to transform relative weight$_{odds}$ into relative weight$_{comp}$ (or vice versa).[13] Consequently, relative weight$_{odds}$ and relative weight$_{comp}$ simply provide two ways of displaying the same information. Runde considers relative weight$_{comp}$ to be the more useful sub-conception, because it is intuitive to think of relative weight as a measure of the completeness of information. Recognize that while the two sub-conceptions of relative weight discussed here parallel statistical odds and probability respectively, relative weight (in any form) remains distinct from probability.

The numerator of relative weight consists in the total amount of known relevant evidence. As such, sheer weight is built into relative weight. Notably, relative weight also incorporates some estimation of the amount of relevant evidence outside our current purview (i.e., ignorance), which many scholars now consider a distinctive feature of Keynesian weight. For example, Ekaterina Svetlova says, "Importantly, the concept of weight allows a meaningful discussion of *gradations of ignorance* and their dynamics" (Svetlova, 2021, 999).

Estimating an amount of relevant ignorance proves to be difficult (if not impossible), but there are at least some potential ways forward. As Runde explains, it is highly plausible to think that we often possess some understanding of the extent of our ignorance regarding a specific question (Runde, 1990, 282). Furthermore, Alberto Feduzi

[13] Specifically, $C_W(c|e) = \frac{W(c|e)}{1+W(c|e)}$.

seems to suggest utilizing subjective assessments of relevant knowledge and ignorance to develop second-order personal probabilities that quantify Keynesian weight (Feduzi, 2010). Alternatively, Brian Weatherson provides an account that uses imprecise (or interval) probabilities to quantify the extent of our ignorance (Weatherson, 2002). We should at least be willing to recognize that estimations of relevant ignorance are sometimes utilized in scientific practice, even if those estimations are rarely made explicit.[14] As a consequence, we should not be too quick to dismiss Runde's relative interpretation of Keynesian weight due to the *prima facie* impracticality of estimating relevant ignorance.

It is natural to assume that acquiring evidence always increases evidential weight. Yet this assumption overlooks cases when new evidence highlights one's own epistemic limitations. In stark contrast to sheer Keynesian weight, relative Keynesian weight allows for weight to *decrease* as a result of gaining new evidence. In particular, Runde notes that the accrual of new evidence can lead to a decrease in relative Keynesian weight if the new evidence causes us to drastically reassess and increase our estimation of ignorance (Runde, 1990, 282-3). Sometimes we learn information that shows we woefully underestimated the true extent of our ignorance on a topic. In those cases, Runde says we can model our increased ignorance via a decrease in Keynesian weight.

We can alter Popper's Paradox of Ideal Evidence to demonstrate an apparent decrease in relative Keynesian weight. Before finding any evidence, the probability of the normal-looking coin landing on heads on the next toss is 0.5. Then, the ideally favorable statistical evidence showed that the coin was tossed 1,000,000 times in the past, with approximately half of those tosses resulting in heads and the other half resulting in tails. Accordingly, the probability of the coin landing on heads on the next toss is updated to 0.5. So far, nothing about the original "paradox" has changed. Now, suppose that we find a second note containing more statistical evidence. This new evidence states that after the initial 1,000,000 tosses, the coin was tossed 2,000,000 more times. Of those 2,000,000 additional tosses, 1,000,000 consecutive tosses resulted in heads, while another 1,000,000 consecutive tosses resulted in tails. The evidence fails to indicate which run of consecutive results occurred first. Our overall evidence describes 3,000,000 tosses, with approximately 1,500,000 tosses resulting in heads and 1,500,000 tosses resulting in tails. Consequently, the probability of the coin landing heads on the next toss remains unchanged at 0.5. As a result of this new evidence, however, I only feel more perplexed about this particular coin.

This case is one in which we can use relative Keynesian weight to model the change in the evidential state. The long strings of consecutive results greatly increase my estimation of my ignorance about this particular coin, which in turn results in a net decrease in relative Keynesian weight. Put differently: the new evidence signals that my evidence about the coin—which I previously considered to be quite substantial—is in fact incomplete. The extreme structure in the new evidence suggests the influence of a systematic but unknown mechanism, thereby undermining my prior confidence in my model of the coin. This increase in perceived model inadequacy reflects a decrease in relative Keynesian weight: I now have strong reason to suspect that my evidence

[14] The Intergovernmental Panel on Climate Change (IPCC) reports seem to be notable cases of explicit scientific attempts to estimate ignorance. Furthermore, the GRADE methodology outlined in Johnston et al. (2019) provides a good example of an attempt to explicitly estimate ignorance in scientific practice.

fails to capture crucial explanatory features of the coin's behavior. After gaining more evidence, I now know that my knowledge of the coin is critically incomplete.

Weatherson provides a case that is meant to serve as another example. Weatherson describes playing a simple type of poker game centered around betting on who has the best hand. As Weatherson says, "Before the bets start, I can work out the chance that some other player, say Monica, has a straight" (Weatherson, 2002, 52). Once betting starts, perhaps Monica's betting affects my probability estimate such that it fluctuates throughout the game but ends up equivalent to the initial estimate. Monica's bets provide additional relevant evidence (i.e., knowledge) that gets incorporated into the relative Keynesian weight I possess about the proposition that she has a straight. Nonetheless, Monica's bets are also intentionally ambiguous, which—provided that Monica's bets operate as she intends—increase my estimation of my own ignorance about her hand being a straight.

Using one of the above formalizations of relative weight to represent these changes makes the evidential effects of Monica's betting clearer. Let k represent my total known evidence relevant to Monica's hand before betting, and let i represent my ignorance of Monica's hand before betting. Using W as our measure of relative Keynesian weight, we have:

$$W(\text{Straight}|k) = \frac{k}{i}.$$

As the game progresses and Monica places her bets, I gain more evidence. Let k' represent my total relevant evidence just before Monica reveals her cards, and let l represent what I learn from Monica's bets throughout the game. As such, my knowledge at this later point of the game is equivalent to the sum (i.e., conjunction) of my knowledge at the beginning of the game and what I learn from Monica's betting:

$$k' = k + l.$$

In addition to increasing my known relevant evidence, Monica's bets also increase my assessment of my own ignorance regarding the cards she possesses. The extent to which Monica's betting increases my ignorance depends on how good of a poker player Monica is. Good poker players create doubt in the minds of others. Assume Monica is an excellent poker player, such that her bets lead to a drastic increase in my estimation of my ignorance. Letting i' represent my new estimate of ignorance and n represent the value of the increase in my ignorance, we have:

$$i \lll i'$$
$$i' = i + n$$
$$W(\text{Straight}|k) = \frac{k}{i} > W(\text{Straight}|k') = \frac{k+l}{i+n}.$$

Of course, this relationship only holds if Monica's betting increases my ignorance by a greater amount than the betting increases my known relevant evidence. In other words, this relationship holds if $n > l$. But since—in this particular example—my estimation of my ignorance increases by a greater amount than the corresponding increase in my knowledge about Monica's hand, Monica's betting leads to a net decrease in the relative Keynesian weight I assign to the proposition that her hand is a straight.

These examples are meant to show that Runde's relative conception of Keynesian weight allows weight to fall in response to gaining evidence. In Runde's hands, Keynesian weight is transformed from a crude measure of the amount of relevant evidence (i.e., sheer weight) into a measure that compares the amount of obtained relevant evidence to an estimate of the amount of not-yet-obtained relevant evidence. This transformation allows Keynesian weight to become more intuitively compelling. As Runde says,

> Failing [the possibility of weight falling with additional evidence], we would have to forego the direct, and intuitive, link between changes in weight and confidence: we would be precluded from saying, quite reasonably, that we have acquired new evidence that reduces our confidence in a previous estimate (Runde, 1990, 283).

In other words, Runde argues that only relative weight can plausibly be linked to rational confidence. Unfortunately, Runde's interpretation conflicts with many of Keynes's own remarks.

3.3 Textual Inconsistencies

Prior to showing the conflicts between Runde's relative conception of weight and Keynes's remarks, I want to first highlight one way in which Runde's relative conception enjoys greater textual support than the sheer conception of weight. As Runde points out, Keynes explicitly makes use of weight in Chapter XXVI of *A Treatise on Probability* (Runde, 1990, 285). There, Keynes says,

> There seems, at any rate, a good deal to be said for the conclusion that, other things being equal, that course of action is preferable which involves least risk, and about the results of which we have the most complete knowledge. In marginal cases, therefore, the coefficients of weight and risk as well as that of probability are relevant to our conclusion. . . . We could, if we liked, define a conventional coefficient c of weight and risk, such as $c = \frac{2pw}{(1+q)(1+w)}$, where w measures the 'weight,' which is equal to unity when $p = 1$ and $w = 1$, and to zero when $p = 0$ or $w = 0$, and has an intermediate value in other cases (Keynes, 1921, 360–1).[15]

Note that p is the probability of a good result, while $q = 1 - p$ is the probability of the complement of that good result; in other words, q is the probability of the good result not occurring. Two main aspects of these passages stand out. First, Keynes seems once again to tie weight to the completeness of our knowledge. Such remarks lend direct textual support to the relative conception of Keynesian weight. Second, Keynes's description of the above coefficient seems to indicate that Keynes intended for weight (at least sometimes) to be indexed to the inclusive interval between zero and one. The only formalization of weight mentioned so far that is likewise indexed is C_W. Consequently, Chapter XXVI lends textual support to a relative conception, and more specifically to C_W.

Although this aspect of the text appears to support Runde's interpretation, numerous other parts of the text clearly conflict with Runde's relative conception of

[15] Keynes's final sentence in this excerpt is ambiguous, but the most charitable interpretation is that he is describing the behavior of the coefficient c, not the variable w. Consider, for instance, the case when $p = 0$ and $w = 1$. That case yields: $c = \frac{2 \cdot 0 \cdot 1}{(1+(1-0)) \times (1+1)} = 0$, which matches his description.

weight. Contrary to the relative conception of weight, Keynes consistently says that weight *must* increase with new relevant evidence. For instance, Keynes says,

> New evidence will sometimes decrease the probability of an argument, but it will *always* increase its "weight" (Keynes, 1921, 78, emphasis added).

Keynes also says,

> If the new evidence is "irrelevant," in the more precise of the two senses defined in §14 of Chapter IV., the weight is left unchanged. If any part of the new evidence is relevant, then the value is increased ... Starting, therefore, with minimum weight, corresponding to *à priori* probability, the evidential weight of an argument rises, though its probability may either rise or fall, with every accession of relevant evidence (Keynes, 1921, 78–9),

as well as,

> If we are to be able to treat "weight" and "relevance" as correlative terms, we must regard evidence as relevant, part of which is favourable and part unfavourable, even if, taken as a whole, it leaves the probability unchanged. With this definition, to say that a new piece of evidence is "relevant" is the same thing as to say that it increases the "weight" of the argument (Keynes, 1921, 79).

Additionally, Keynes's second rule for comparing weights implies that weight always increases with increases in relevant evidence (Keynes, 1921, 80). Keynes is repeatedly clear on this point—monotonicity is a fundamental feature of weight. Keynes reiterates the point later in Chapter VI:

> The fundamental distinction of this chapter may be briefly repeated. One argument has more *weight* than another if it is based upon a greater amount of relevant evidence; but it is not always, or even generally, possible to say that of two sets of propositions that one embodies *more* evidence than the other. It has a greater *probability* than another if the balance in its favour, of what evidence there is, is greater than the balance in favour of the argument with which we compare it; but it is not always, or even generally, possible to say that the balance in one case is greater than the balance in the other. The weight, to speak metaphorically, measures the *sum* of the favourable and unfavourable evidence, the probability measures the *difference* (Keynes, 1921, 85).

Throughout his primary explication of weight, then, Keynes seems emphatic about weight increasing with increases in relevant evidence. Accordingly, Runde's interpretation conflicts with a large swath of the textual evidence. Indeed, Runde acknowledges this issue by noting that his interpretation "contradicts many of Keynes's other statements" (Runde, 1990, 283).

Most scholars working prior to Runde's 1990 paper assumed something akin to the sheer conception of Keynesian weight. Nonetheless, Runde argues that relative weight "is the measure to use, if, as Keynes does, it is to be linked to investor confidence" (Runde, 1990, 283). Similarly, Weatherson adopts a relative conception of Keynesian weight in order to strengthen the tie between weight and Keynes's comments on uncertainty in Keynes's economic magnum opus, namely *The General Theory of Employment, Interest, and Money* (Weatherson, 2002, 52). For my money, it seems like there is a compelling case to be made that Keynesian weight is only worthwhile if it can be linked to rational confidence. As we have seen from Runde, only relative weight—and not sheer weight—can potentially account for rational confidence.[16] So we face a dilemma. The sheer conception of Keynesian weight possesses strong textual support but is intuitively unsatisfactory, while the relative conception of Keynesian weight is intuitively intriguing yet textually unsatisfactory. My interpretation overcomes this dilemma.

4 Reconciling Interpretations of Keynesian Weight

My aim in this section is to demonstrate how to reconcile the monotonicity of sheer weight with the relative conception of Keynesian weight. I proceed by first explaining how a reconsideration of "ignorance" provides the key to the reconciliation. Such a reconsideration allows for a distinction between estimated and actual weight, both of which are forms of relative Keynesian weight. Next, I use former United States Secretary of Defense Donald Rumsfeld's (somewhat infamous) distinction between known unknowns and unknown unknowns to highlight differences between estimated and actual Keynesian weight. The section concludes with some brief remarks about future research on Keynesian weight.

4.1 On Ignorance

Runde's relative conception of Keynesian weight can be made consistent with Keynes's remarks about weight always increasing. The key to the reconciliation is to reconsider what exactly is meant by "ignorance" when it is used as a term in W or C_W. Given a different—yet plausible—characterization of ignorance, even relative Keynesian weight must increase with evidential gains.

For Runde, the I term in W and C_W clearly refers to a *personal estimate* of the amount of relevant ignorance. As a case in point, Runde says, "But it is surely possible, in principle, that we may sometimes learn something that leads us to drastically *reassess* I, to revise it upward by more than any increase in K" (Runde, 1990, 282–3, emphasis added). After discussing examples from Keynes that appear to lend support to his interpretation, Runde goes on to say, "These examples seem to bear the consequence that if new evidence is acquired to the effect that there are more alternatives or a larger field of possibility than previously *imagined*, then weight can fall despite our knowledge increasing" (Runde, 1990, 283, emphasis added). Previously, we saw that relative Keynesian weight (under Runde's interpretation) sometimes decreases in response to new relevant evidence precisely because new relevant evidence can cause a reasoner to drastically increase their *estimate* of ignorance.

If, however, we interpret the I term in the above formalizations to refer to something like "the *actual* amount of the agent's relevant ignorance as judged from an omniscient perspective," then relative Keynesian weight increases monotonically with the accrual of new evidence. On this interpretation, gaining new relevant evidence simply involves shifting relevant evidence from the I term of the expression to the K term of the expression. Since I is in the denominator and K is in the numerator, any

[16] An anonymous reviewer mentioned that even relative weight may be too crude for measuring rational confidence. Relative weight, after all, seems to fail to account for the quality of evidence.

shift of evidence from I to K results in an overall increase in Keynesian weight. We can also think of this interpretation as stating that the value of the denominator of C_W remains constant over time, and that new relevant evidence simply increases the numerator, K, relative to the constant denominator, $(K + I)$.

Since the denominator of C_W remains constant over time, the total amount of relevant information (i.e., $(K + I)$) is an unchanging quantity that exists independently of our estimations of ignorance. With that in mind, we can now see a peculiar feature of Runde's interpretation. Because Runde takes the amount of total relevant information (i.e., $(K + I)$) to be a function of estimations of ignorance, the amount of total relevant information must change as estimations of ignorance change. Put differently, Runde's interpretation makes it possible for an agent to alter the total amount of information that is relevant to an argument simply by reevaluating the extent of their own ignorance with respect to that argument. Surely that cannot be right; the amount of total relevant information is what it is, regardless of what anyone may think of it. By treating ignorance as an objective measure, my interpretation keeps the amount of total relevant information constant despite any potential fluctuations in the estimated amount of ignorance. Unlike Runde's interpretation, my interpretation ensures that knowledge becomes more complete with evidential gains, while the total quantity of relevant information remains constant. Evidential gains simply involve reallocating relevant information from the pool of relevant ignorance to the pool of relevant knowledge.

An example may make the differences between these interpretations more concrete.[17] Consider an agent who possesses some formal, academic training in economic theory. Upon the completion of their training, the agent possesses a rational belief regarding a future increase in the rate of inflation. The agent's rational belief is based entirely upon their knowledge of one economic factor that they know influences the rate of inflation. Put differently: the agent thinks the rate of inflation is completely explained by changes in a single economic variable. The agent is unaware of other economic variables that influence inflation rates, and they are unaware that they misunderstand inflation rates in this regard.

Both my interpretation and Runde's interpretation consider the weight of the argument that justifies this agent's rational belief to be the ratio of relevant knowledge (i.e., K) to total relevant information (i.e., $K + I$). Our interpretations differ, however, with respect to whether this particular agent's belief possesses a high Keynesian weight or a low Keynesian weight. According to my interpretation, the agent's belief possesses a low Keynesian weight because the agent's actual ignorance with respect to inflation rates is vast. While the agent knows about one economic variable that influences the rate of inflation, that knowledge is miniscule in comparison to the actual total relevant information regarding the rate of inflation. Runde's interpretation, in contrast, requires the Keynesian weight of this agent's belief to be high. This agent believes they have a complete explanation of changes in inflation rates, which implies that the agent estimates their ignorance with respect to this topic to be low (or even

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[17] I am grateful to an anonymous reviewer for suggesting this example.

null). Consequently, the ratio of total knowledge to estimated total relevant information is high. Treating ignorance as an actual, objective measure thus results in a substantively different conception of Keynesian weight than Runde's interpretation.

There exists one way to reconcile all the distinct considerations presented so far. Namely, we should interpret Keynesian weight as C_W, with the term for relevant ignorance (I) defined as the actual amount of relevant evidence that is unknown to the agent, as judged from an omniscient perspective. The formalizations of C_W and W given above do not appear in Keynes's work, but rather are derived from the text by Runde. As shown above, C_W can be put into terms of W through a simple mathematical transformation, which rectifies the parts of the text that separately favor C_W and W. Moreover, C_W is consistent with Keynes's remarks regarding the completeness of knowledge. Since C_W is indexed to the inclusive range between zero and one, C_W aligns with Keynes's discussion of the coefficient of weight and risk. Note that, because the total amount of relevant information (i.e., $K + I$) varies between arguments, it will often be impossible to compare the relative weight of arguments—even when those arguments are based on the same exact set of relevant evidence (i.e., K). Finally, by interpreting "relevant ignorance" as the actual amount of relevant, unknown evidence judged from an omniscient perspective, we can bring C_W into line with Keynes's remarks about weight always increasing with increases in relevant evidence.

Simply put, my interpretation allows us to maintain the relative weight conception as presented by Runde in W and C_W, while also overcoming the textual inconsistencies presented in 3.3 above. According to my interpretation, it is no accident that Keynes uses the phrasing "the *absolute* amounts of relevant knowledge and of relevant ignorance respectively" when he introduces relative weight (Keynes, 1921, 78, emphasis added). At least in *A Treatise on Probability*, Keynes focuses on actual amounts of knowledge and ignorance rather than personal estimates of our knowledge and ignorance. The interpretation presented here better aligns with all relevant aspects of the source material.

4.2 Estimated and Actual Weight

Rumsfeld's distinction between known unknowns and unknown unknowns can further clarify the fundamental difference between Runde's interpretation and the interpretation presented in this paper. Faulkner, Feduzi, and Runde describe Rumsfeld's distinction by saying,

> While there may be many features of the world of which an individual is ignorant, there are likely some (the known unknowns) that she knows she is ignorant of while there are others (the unknown unknowns) that she does not even know she does not know (Faulkner et al., 2017, 1283).

Here, unknown relevant information is divided into two types. If we use μ_k to represent the amount of known unknown relevant information and μ_u to represent the amount of unknown unknown relevant information, then we can use the following expression to represent the total amount of unknown relevant information U:

$$U = \mu_k + \mu_u.$$[18]

Some terminological clarification is in order. In Rumsfeld's distinction, the adjectives "known" and "unknown" operating on the noun "unknowns" are clearly epistemic in nature. That is to say that those adjectives deal with an agent's knowledge about what they do not know. Given the way the word "knowledge" is used throughout this literature, I treat "known unknowns" (i.e., μ_k) as a personal estimate of ignorance. Such a treatment requires us to make "unknown unknowns" (i.e., μ_u) into an error term that ensures the total amount of unknown relevant information (i.e., U) is accurate.

With those terminological clarifications in place, we can now see the benefit of introducing Rumsfeld's distinction. According to my interpretation of Keynesian weight, $I = U$ in C_W and W. That is to say, my interpretation states that relevant ignorance is identical to the total amount of unknown relevant information as judged from an omniscient perspective—namely, the sum of known unknowns and unknown unknowns. Based on Runde's frequent references to personal estimations of ignorance, it makes sense to read his interpretation as stating that $I = \mu_k$ in C_W and W. As such, Runde's interpretation states that relevant ignorance is identical to the agent's known unknowns. In terms of the notation specified so far in this paper, my interpretation of Keynesian weight is:

$$C_W(c|e) = \frac{K}{(K+U)},$$

while Runde's interpretation Keynesian weight is:

$$C_W(c|e) = \frac{K}{(K+\mu_k)}.$$

Stated another way, the difference between these interpretations is the consideration of the "unknown unknowns" (i.e., μ_u). Due to this difference, we might think of my interpretation of Keynes as an expression of *actual Keynesian weight*, while Runde's interpretation of Keynes is an expression of *estimated Keynesian weight*.

Reconsider the example of the agent with economic training. In that example, the agent knows about a connection between some single economic variable and inflation, which causes them to hold a rational belief regarding a future increase in inflation rates. Furthermore, the agent believes that the connection between the economic variable and inflation completely explains changes in inflation. Nevertheless, the phenomenon of inflation is more complex than this agent takes it to be, which is to say that other economic variables also affect the rate of inflation. Crucially, the agent is unaware of the connection between inflation and these other economic variables. Although the agent does not estimate their ignorance to be significant, their actual ignorance with respect to this belief is large. For the sake of simplicity, assume that the agent estimates their ignorance with respect to inflation to be zero.

[18]While this expression and the subsequent formalism may seem excessive, the formalism presented here further clarifies the differences between my interpretation and Runde's interpretation. Some readers may wish to skip to the paragraph that begins, "Relatedly, Rumsfeld's distinction...," below.

In terms of Rumsfeld's distinction, then, the agent's known-unknown value (i.e., μ_k) is null, while their unknown-unknown value (i.e., μ_u) is substantial. Let k represent the agent's knowledge about inflation rates. Thus, we have:

$$0 = \mu_k < k \lll \mu_u.$$

When this relationship is considered in light of the above interpretations of relative Keynesian weight, it becomes clear why Runde's interpretation dictates that the Keynesian weight of this agent's belief is 1, while my interpretation dictates that the Keynesian weight of this agent's belief is less than 1. Put formally, the actual Keynesian weight (i.e., C_W^A) of this agent's belief is:[19]

$$0 < C_W^A = \frac{k}{(k + \mu_k + \mu_u)} < 1,$$

while the estimated Keynesian weight (i.e., C_W^E) of this agent's belief is:

$$C_W^E = \frac{k}{(k + \mu_k)} = 1.$$

Now suppose the agent becomes employed at a central bank. Due to the agent's new employment, they become aware of their misunderstanding of inflation rates. Through this newfound awareness, the agent learned something about the inflation phenomenon. Namely, the agent learned about connections between the rate of inflation and other economic variables. Let $k + l$ represent the agent's known relevant evidence after they become aware of their misunderstanding. The new relevant evidence, l, is revealing in two ways. First, it taught the agent something about inflation, namely that there are more economic variables that affect rates of inflation. Second, the relevant evidence taught the agent something about themselves, namely that they previously underestimated their own ignorance with respect to inflation. While the information about the agent is not relevant evidence regarding inflation, the information about the agent still affects the estimated Keynesian weight of the agent's belief. This second change is represented by an increase in the agent's estimated ignorance with respect to inflation. After working at the central bank, the agent realizes that there is some relevant evidence, η, which they lack. Provided that μ_k' represents the agent's new known unknowns, the following relationships exist:

$$\mu_k' = \eta$$
$$0 = \mu_k \lll \mu_k'.$$

Given these facts, the estimated Keynesian weight of the agent's belief regarding inflation decreases after they begin working at the central bank.[20] In contrast, the actual Keynesian weight of the agent's belief *increases* after they begin working at the

[19] For the sake of brevity, I dropped the references to the argument in this example.

[20] Precisely: $C_W^{E'} = \frac{k+l}{([k+l]+\eta)} < C_W^E = \frac{k}{(k+0)} = 1$.

central bank. Let μ'_u represent the agent's unknown unknowns after beginning their work at the central bank. Then:

$$\mu'_u = \mu_u - \eta - l$$
$$C_W^{a'} = \frac{k+l}{([k+l] + \eta + [\mu_u - \eta - l])}$$
$$C_W^{a'} = \frac{k+l}{(k+\mu_u)} > C_W^A = \frac{k}{(k+\mu_u)}.$$

Thus, the actual Keynesian weight of the agent's belief increases; the agent learned about other real connections between economic variables and inflation rates. Learning this information increased the agent's known relevant evidence in comparison to the (actual) total relevant information regarding inflation rates. This example demonstrates the precise way in which the denominator of actual Keynesian weight remains constant as agents learn new information: while the learned information l is subtracted from the agent's ignorance, the learned information l is simultaneously added to the agent's knowledge. Likewise, the agent's new estimation of their ignorance η is subtracted from their previous unknown unknowns, but it is simultaneously added to the denominator via the μ_k term. As a result of the denominator remaining constant while learning, evidential gains must increase actual Keynesian weight.

Relatedly, Rumsfeld's distinction also helps us see why one might choose to focus on estimates of ignorance (as Runde does) when contemplating relative Keynesian weight. As difficult as it might be to provide a personal estimation of relevant ignorance, such an estimation is far easier to obtain than a measure such as U. There is *prima facie* reason to think that it is impractical to obtain a justified measure of μ_u and thus U. In fact, I think that undersells the case. Depending on how one interprets the phrase "unknown unknowns," it might be—by definition—*impossible* to obtain a justified measure of μ_u. As a result, accurately measuring U appears intractable.

In other words, Runde's interpretation of Keynesian weight gets something importantly right. Specifically, Runde gets the metaphysics of Keynesian weight wrong while getting the epistemology of Keynesian weight roughly correct. By "epistemology of Keynesian weight," I mean what we—as limited, fallible beings—can know (or at least justifiably believe) about Keynesian weight. The epistemology of Keynesian weight concerns how we can make justified estimates of Keynesian weight. By "metaphysics of Keynesian weight," I mean the actual value of Keynesian weight, which is independent of any personal estimates of our ignorance or knowledge. Regardless of any personal estimates of ignorance or knowledge, there are facts about what we know and do not know. The metaphysics of Keynesian weight concerns these facts.

The conflation of the metaphysics and epistemology of Keynesian weight is the fundamental source of Keynes's seemingly confused remarks. From an omniscient perspective, the completeness of an agent's relevant knowledge (i.e., actual Keynesian weight) increases monotonically as the agent gains more relevant evidence. Nonetheless, the agent's estimation of their ignorance is constrained by their limited perspective. As such, the agent's estimated Keynesian weight can rise or fall (as Runde describes) as new relevant evidence forces the agent to reassess their estimation of relevant ignorance. By distinguishing between estimated and actual Keynesian weight, my account subsumes Runde's interpretation while resolving the textual inconsistencies caused by that interpretation.

Runde's interpretation of Keynes provides a compelling (albeit preliminary) account of the epistemology of Keynesian weight. For that reason, it would be a mistake to reject the interpretation wholesale. Instead, the interpretation should be put into proper perspective. Estimated Keynesian weight concerns what fallible agents can know about the completeness of their knowledge, which makes it an obvious entry point for applying Keynesian weight to real-world situations.[21] Although the omniscient perspective makes measuring actual Keynesian weight impractical for human minds, we can nonetheless *estimate* the completeness of our knowledge with respect to particular topics. Put differently: Runde's interpretation illuminates the path toward making Keynesian weight applicable, while actual Keynesian weight is what Keynes meant when writing *A Treatise on Probability*. It is unlikely that actual Keynesian weight can be operationalized, yet clarifying the nature of actual Keynesian weight in contrast to estimated Keynesian weight should help researchers focus their attempts to apply Keynesian weight. Actual Keynesian weight is a concept of the past; estimated Keynesian weight is a concept for the future.

Keynes's conception of probability leaves room for agents to misjudge the actual value of a probability relation. On my view, something similar holds for Keynesian weight. Actual Keynesian weight is what it is, regardless of our estimations of ignorance (or knowledge, for that matter). To put it another way, we can be incorrect about Keynesian weight, and our estimates of Keynesian weight might sometimes behave non-monotonically, as shown by the cases when our estimates of total ignorance increase with new relevant evidence. In its essence, however, Keynesian weight is indeed monotonic in the sense repeatedly emphasized by Keynes. Nonetheless, when it comes time to apply Keynesian weight, we have no option but to focus on our *estimates* of relevant knowledge and ignorance, respectively. As such, future research should focus on obtaining justified estimates of ignorance (i.e., μ_k) while maintaining a clear-minded recognition that knowledge of the *actual* amount of relevant unknown evidence (i.e., U) is out-of-reach for epistemically limited beings such as us.

To that end, I think there are at least three clear next steps for research on Keynesian weight. First, there are some existent problems with defining "relevant evidence"(Cohen, 1986; O'Donnell, 1992; Weatherson, 2002). Given the interpretive shift presented in this paper, it is worth considering how (if at all) these existing problems are affected. Second, Keynes noted fundamental differences between probable error and Keynesian weight. In contrast, Peirce argued that probable error captured Keynesian-weight-like considerations (Kasser, 2016; Peirce, 1992). It may indeed be impossible to capture actual Keynesian weight via a probabilistic measure such as probable error, but might we use a probabilistic measure for adequately estimating Keynesian weight? In other words, can the fundamental distinction between weight and probability be

[21] Furthermore, there are clear connections between estimated Keynesian weight and recent work on *unawareness*. See Schipper (2014) for more on unawareness, and see Svetlova (2021) for some discussion of Keynesian weight in the context of the unawareness literature. I am grateful to an anonymous reviewer for suggesting this link.

relaxed when we recognize that we are simply developing an estimate of Keynesian weight? For his part, Keynes himself suggested that probable error might serve as a practical proxy for weight (Keynes, 1921, 82).[22] Finally, I suspect that the problems associated with defining relevant evidence are rooted in a more fundamental problem of counting evidence. How do we accurately "count" our evidence? Clearly, any thorough operationalization of actual Keynesian weight would need to address the problem of counting evidence. If, however, we can use a probabilistic measure to estimate Keynesian weight, then it may be possible to bypass the issue altogether. Probabilistic tools like probable error do not directly rely on counting discrete pieces of evidence. Accordingly, if probable error can serve as a way to estimate Keynesian weight (as both Keynes and Peirce seemed to think it might), then the vast literature on probabilistic estimation might provide a ready-made scaffold for addressing weight without solving the difficult problem of counting evidence.

5 Conclusion

Ultimately, we are left with the following revelations. First, Keynesian weight (as specified by Keynes himself) is best interpreted as the version of relative Keynesian weight found in C_W. Second, when reading C_W, K should be read as "the actual amount of known relevant evidence," while I should be read as "the actual amount of unknown relevant evidence as judged from an omniscient perspective," or equivalently, "the sum of relevant known unknowns and relevant unknown unknowns." Finally, for future research on Keynesian weight to be productive, researchers must delineate actual Keynesian weight from our fallible estimates of this objective property.

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Declarations

Not applicable.

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[22] Keynes specifically says, "The connection between probable error and weight, such as it is, is due to the fact that in scientific problems a large probable error is not uncommonly due to a great lack of evidence, and that as the available evidence increases there is a tendency for the probable error to diminish. In these cases, *the probable error may conceivably be a good practical measure of the weight.*" (Keynes, 1921, 82, emphasis added).

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